International Journal of Research in Applied Science
Vol.3 Issue.1 (November–December) 2016

A NEW METHOD FOR EVALUATION OF FUZZY RELIABILITY OF MULTISTAGE INTERCONNECTION NETWORKS

Deepak Kumar Panda
Department of Master in Computer Application,
Gandhi Institute for Technological Advancement (GITA), Bhubaneswar-752054 Odisha, India

Received on 18th June 2016 and Accepted on 31st July 2016

ABSTRACT

This paper presents a new and simple method for evaluating the fuzzy reliability of multistage interconnection networks (MIN). A new algorithm to evaluate the fuzzy reliability has been proposed. The algorithm enumerates all the path sets from the source node to destination node. Then the system fuzzy reliability is expressed in terms of fuzzy probability of the disjoint terms of all path sets. The proposed method only uses two operations i.e. multiplication and complementation in evaluating system reliability. The proposed method is well illustrated through a simple example. Fuzzy reliability of some important fault-tolerant multistage-interconnection networks have been evaluated by the proposed method.

INDEX TERMS – Reliability, Fuzzy set, Interconnection network

NOTATIONS

X	a set containing a space of points in the probability domain
x	an element of X
p_i	fuzzy probability of an event i
$\overline{p}_i$	complement of fuzzy probability of an event i
$\mu_{p_i}(p)$	membership function of fuzzy probability p_i
N	number of nodes of the MIN
R	fuzzy reliability of MIN
G	reliability logic graph
V	vertex set
E	edge set
S	system success containing all paths between the source node(s) to destination node (t)
P_i	path at the i^{th} step
$W, \overline{W}$	indicator variables

Assumptions

1. Initially, all components of the system are in good conditions.
2. The link failure and link success probability is assumed to be fuzzy numbers
3. Failures cannot be determined with certainty.
4. Repair facility is not available.

I. INTRODUCTION

The computation of reliability is of paramount importance in parallel processing environments where thousands of processors cooperate with each other to solve a complicated problem. However, there lies a large degree of uncertainty in system failure and therefore, the conventional methods of reliability evaluation for large parallel computer system may not be appropriate to get a realistic value. Under such condition, one of the tools to cope with imprecision of available information in reliability analysis is *fuzzy set theory* [1], which is based on uncertainties like vagueness, ambiguity and imprecision.

Fuzzy set theory was first introduced by Zadeh [1]. A fuzzy set can be defined as follows:

Let X be a space of points and an element of X be denoted by x, i.e. X={x}. A fuzzy set A in X is characterized by a membership function $\mu_A(x)$, which is a real number in the interval [0,1] and represents the degree of membership of x in A.

In the conventional methods [2-4], it is required to find the minimized expression of system reliability using Boolean algebra. However, these expressions cannot be used in fuzzy set theory because of non-applicability of complementary laws. The expression used for fuzzy reliability of parallel systems has to be different from the expression of conventional probability analysis for obvious reason.

Keller and Kara-Zaitri [5] presented a method for assessment of reliability of a non-series parallel network using fuzzy logic. Soman and Misra analysed fuzzy fault tree using resolution identity [6], Tanaka et al [7] and Misra and Weber [8] showed how fuzzification can be carried out for the quantitative analysis of fault tree. Chowdhury and Mishra [9] evaluated the reliability of a non-series parallel network. Bastani et al [10] considered the reliability modeling continuous process-control system. Patra et al [11] presents a method for evaluating fuzzy reliability of a communication network with fuzzy element capacities and probabilities. But none of methods discussed above considers the multistage interconnection networks and suggests a general method of evaluating fuzzy reliability of multistage interconnection networks where there lies a large degree of uncertainty in system failure. Tripathy et al [12] have proposed a method to evaluate fuzzy reliability of MINs. However, the method is not general and cannot be applied as such for all MINs. So, there is always a need to search for a general and efficient method to evaluate the fuzzy reliability of such systems.

The main objective of this paper is to explain how the fuzzy set concepts can be applied in evaluating the system reliability. In this paper, a general and efficient method has been proposed to find an expression of fuzzy system reliability of parallel systems taking in to consideration the special requirements of fuzzy sets.

II. CONCEPT OF FUZZY PROBABILITY

Fuzzy probability represents a fuzzy number between zero and one, assigned to the probability of an event. One can chose different types of membership functions for fuzzy probability. For instance, a fuzzy probability may have a trapezoidal membership function. The fuzzy probabilities of an event i can then be denoted by a four parameter function i.e.

Published by R&D Cell, Gandhi Institute for Technological Advancement, Bhubaneswar-752054

$$p_i = (\alpha_{i1}, \alpha_{i2}, \beta_{i2}, \beta_{i1}) \qquad (1)$$

The membership function is given by

$$\mu_{p_i}(p) = \begin{cases} 0, & 0 \le p \le \alpha_{i1} \\ 1 - \dfrac{\alpha_{i2} - p}{\alpha_{i2} - \alpha_{i1}} & \alpha_{i1} \le p \le \alpha_{i2} \\ 1 & \alpha_{i2} \le p \le \beta_{i2} \\ 1 - \dfrac{p - \beta_{i2}}{\beta_{i1} - \beta_{i2}} & \beta_{i2} \le p \le \beta_{i1} \\ 0 & \beta_{i1} \le p \le 1 \end{cases} \qquad (2)$$

Operation used in computing fuzzy reliability

Let p_i and p_j be two fuzzy sets that have membership functions given by $\mu(p_i)$ & $\mu(p_j)$, respectively. The operations used in fuzzy reliability evaluation, i.e. multiplication and complementation can be defined as follows:

1. *Multiplication-*

$p_i.p_j$ = *product of* p_i *and* p_j

$$= \mu_{p_i p_j}(p) = \mu_{p_i}(p).\mu_{p_j}(p) \qquad (3)$$

How ever, Tanka et al [8] provided an approximation of the multiplication procedure by defining

$$p_{ij} = p_i . p_j = (\alpha_{i1}.\alpha_{j2}, \alpha_{i2}.\alpha_{j2}, \beta_{i2}.\beta_{j2}, \beta_{i1}\beta_{j1}) \qquad (4)$$

2. *Complementation*

The complementation of any fuzzy set p_i will be given by

$$\overline{p}_i = 1 - \mu_{p_i} \qquad (5)$$

for example, in case of trapezoidal membership function, one could obtain

$$\overline{p}_i = (1 - \alpha_{i1}, 1 - \alpha_{i2}, 1 - \beta_{i2}, 1 - \beta_{i1}) \qquad (6)$$

III. PROPOSED METHOD FOR FUZZY RELIABILITY EVALUATION

First, the multistage interconnection network is converted into the equivalent reliability logic graph G{V,E}, where V is the vertex set and E is edge set. The edge (link) success and edge failure probability is assumed to be fuzzy numbers. Let Pi be the ith path generated from the given source (s) to the given destination (t). Let S be the union of all the paths generated from the source (s) to destination (t). The system success S on disjointing gives (S)dis. Fuzzy reliability can then be obtained on replacing all indicator variables by their fuzzy probabilities and logical sum and product operator by their fuzzy arithmetic counterparts.

$$R = (S_{dis})_{\{W_i, \overline{W}_i, \cup, \cap\} \to \{p_i, q_i, +, .\}} \qquad (7)$$

Proposed algorithm

1. *Convert the multistage interconnection network to a reliability logic graph with V vertices and E set with source(s) and destination (t) node.*
2. *Generate Trapezoidal membership functions for each edge e* $\in E$ *of the graph.*
3. $S = \phi$, *i= 1;*
4. *while (* P_i *not a cycle and the end points u, v* $\in P_i$ *are s and t)*

   {

Generate the Path P_i

$$S = S \cup P_i$$

Next P_i

i=i+1;

}

5. *repeat steps 6-7 for i =1 to 4*
6. *Find $(S_i)_{dis}$ by the edge success and edge failure of each edge in S as i^{th} parameter of the membership function and i^{th} parameter of membership function in its complement form.*
7. *The system fuzzy reliability is then expressed as*

$$R_i = (S_i)_{dis\{\cup,\cap\rightarrow+,\times\}}$$

IV. Illustration

The proposed method is illustrated through the following example

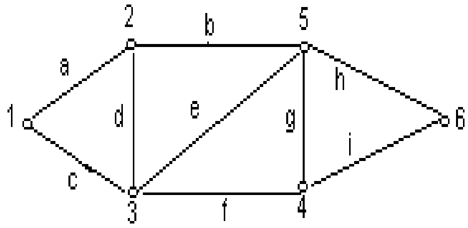

Figure 1. A network of 6 nodes and 9 links

Assuming the trapezoidal membership function as given in Equation 2, the fuzzy probabilities of links *a, b, c, d, e, f, g, h, i* of the network in Fig.1 are given as follows

p_a *=(0.22, 0.8, 0.95, 0.99);*

p_b *=(0.11, 0.7, 0.88, 0.96);*

p_c *=(0.35, 0.85, 0.91, 0.98);*

p_d *=(0.1, 0.42, 0.65, 0.93);*

p_e *=(0.17, 0.6, 0.83, 0.96);*

p_f *=(0.3, 0.5, 0.7, 0.90);*

p_g*=(0.10,.76,0.85,0.97);*

p_h*=(0.1,0.64,0.7,0.9);*

p_i*=(0.1,0.55,0.60.9);*

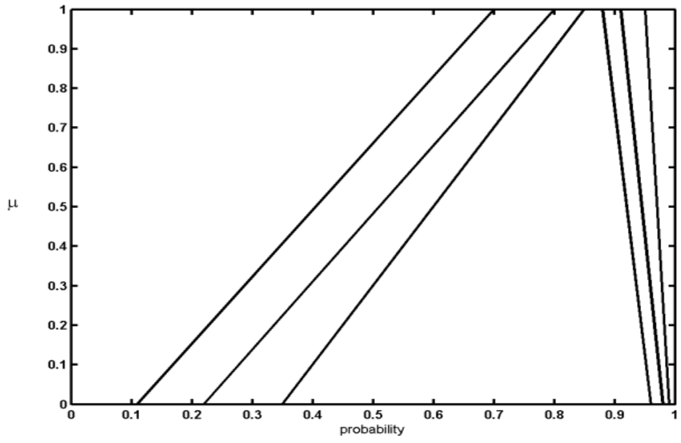

Figure 2. Fuzzy probabilities of links a 1

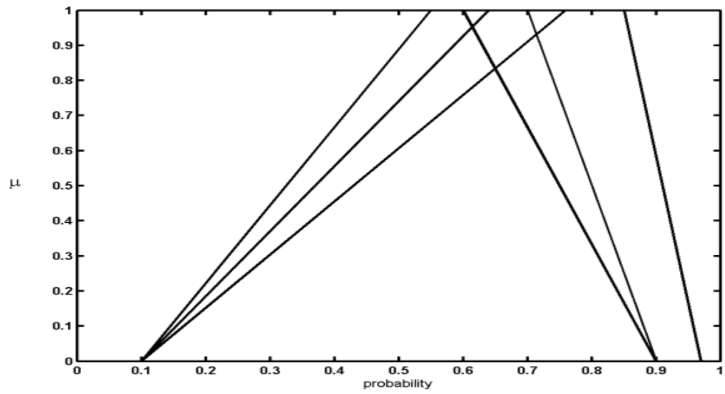

Figure 3. Fuzzy probabilities of links d,e,f

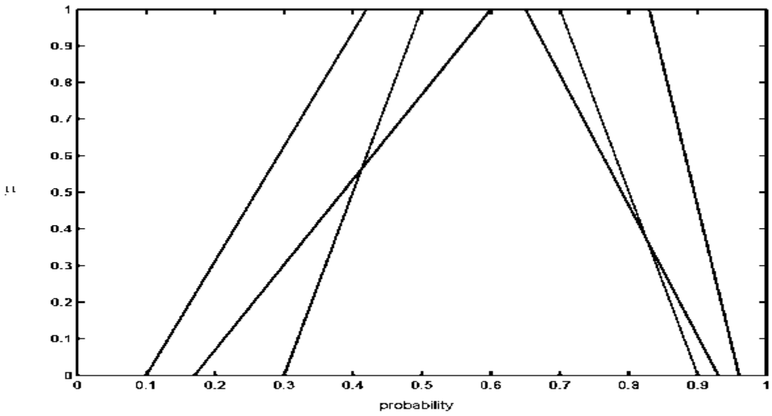

Figure 4. Fuzzy probabilities of links g,h,i

The path set is given by

P={abh, adeh,adfi,ceh,cfi,cegi, cfgh}

Using the proposed algorithm, the membership function of the fuzzy reliability of the network is given by

R= {0.4306, 0.5476, 0.9024, 0.9940}

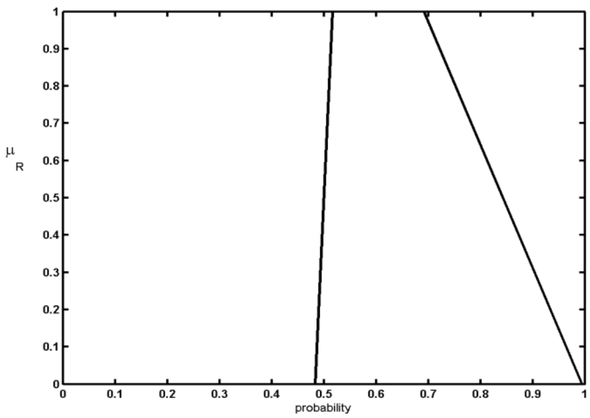

Figure 5. Fuzzy reliability of the network with 6 nodes and 9 edges

V. RESULTS AND DISCUSSIONS

The fuzzy reliability of the three important fault-tolerant multi stage interconnection networks [13] viz. Extra Stage Cube (ESC), Extra Stage Shuffle Exchange Network (ESEN) and Multipath Omega Network

(MPON) have been evaluated by the proposed approach. The membership functions of the said multistage interconnection networks are plotted against the probability (Figs. 6-8). The parameter functions of the said MINs are presented in Table 1, the inference that can be drawn from Table 1 is that the fuzzy reliability of ESC, ESSE and MON lie between the limit 0.58-0.70, 0.55-0.70 and 0.56-0.70 respectively with a 100% possibility which indicates ESC has a better fuzzy reliability than ESEN and MPON.

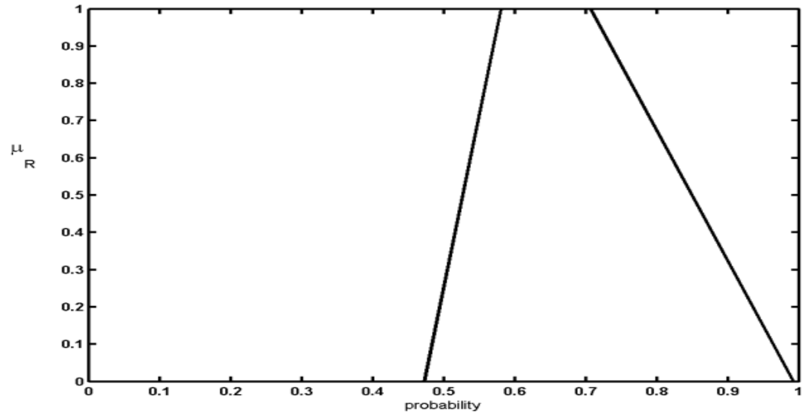

Figure 6. Fuzzy reliability of Extra stage cube of size (8 x 8)

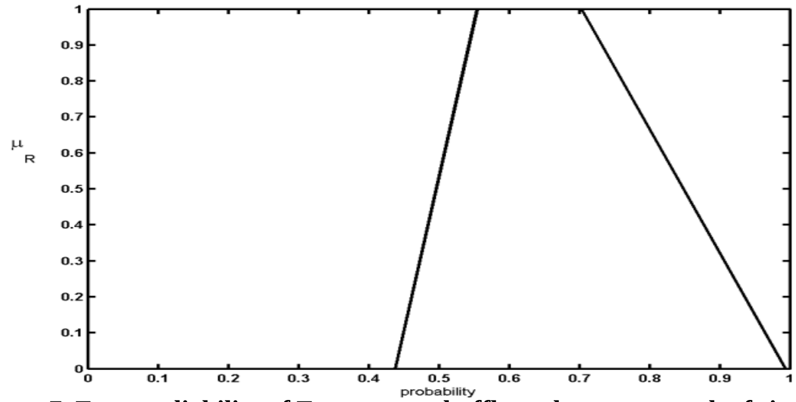

Figure 7. Fuzzy reliability of Extra stage shuffle exchange network of size (8 x 8)

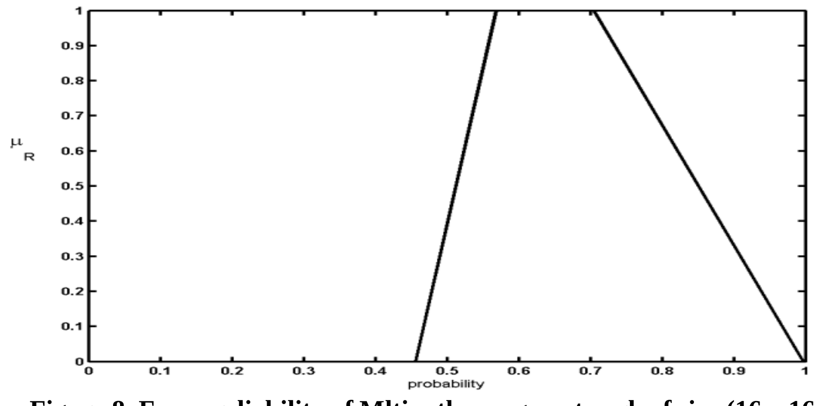

Figure 8. Fuzzy reliability of Mltipath omega network of size (16 x 16)

Table 1. Parameter functions of the multi stage interconnection networks (MINs)

MIN	α_1	α_2	β_2	β_1
ESC	0.4729	0.5812	0.7068	0.9920
ESSE	0.4374	0.5544	0.7028	0.9934
MON	0.4556	0.5684	0.7046	0.9965

Published by R&D Cell, Gandhi Institute for Technological Advancement, Bhubaneswar-752054

VI. CONCLUSION

In a parallel computing environment there lies large degree of uncertainty in system failure and therefore, conventional methods of reliability evaluation may not be appropriate to get a realistic value. Under such situations it is most appropriate to use the concept of fuzzy set. Fuzzy concepts are discussed in the background of this chapter. The importance of fuzzy reliability and its evaluation methods have been presented. Basically, the proposed methods use the path enumeration technique in evaluating fuzzy reliability. The link failure and link success probability is assumed to be fuzzy numbers. In this paper, a general and efficient method to analyze the fuzzy reliability of MINs has been proposed. The method is followed by mathematical basis, algorithm and illustrations. Results have been obtained for three classes of important fault-tolerant MINs.

REFERENCES

[1] L.A. Zadeh, "Fuzzy sets", *Information control*, vol. 1, no.1, pp. 338-353, 1965.

[2] Ali M. Rushdi, King Abdul Aziz, " On reliability evaluation by network decomposition", *IEEE Transactions on Reliability,* vol.R-33, no.5, pp.379-384, Dec 1984.

[3] J.A. Abraham, "An Improved Algorithm for Network Reliability", *IEEE Transactions on Reliability,* vol R-28, no. 1, pp.58-61, 1979.

[4] C.R. Tripathy, R.N. Mahapatra, R.B.Mishra, "Reliability Analysis of Hypercube Multicomputer", *Microelectronics & Reliability An International Journal*, vol.37 no.6, pp.885-891, 1997.

[5] A.Z. Keller and C. Kara-Zaitri, "Further Applications of fuzzy logic to Reliability Assessment and Safety Analysis*", Microelectronics & Reliability-An International Journal,* 29, 399-404, 1989.

[6] K. P. Soman and K. B. Misra, "Fuzzy-tree analysis using resolution identity", *Journal of Fuzzy Math.*, vol. 1, pp. 193-212, 1993.

[7] H. Tanaka, L. T. Fan, F. S. Lai and K. Toguchi, "Fault tree analysis by fuzzy probability", *IEEE Transaction on Computer,* vol. 32, pp.455-457, 1983.

[8] K. B. Misra and G. G. Weber, Use of fuzzy set theory for level-I studies in probabilistic risk assessment, *Fuzzy Sets and systems*, 37, 139-160, 1990.

[9] S. G. Chowdhury and K.B. Mishra, "Evaluation or fuzzy reliability of a non-series parallel network", *Microelectronics & Reliability*, vol.32, pp.1-4 1992.

[10] F.B. Bastani, I.R. Chen and T. Tsao, "Reliability of systems with fuzzy-failure criterion", *1994 Proceeding of Annual Reliability and Maintenance. Symposium*, 1994, pp.442-448.

[11] S. Patra et. al., "Reliability evaluation of flow networks considering multistate modeling of network elements", *Microelectronics & Reliability*, vol.33(14), pp. 2161-2164, 1993.

[12] C.R. Tripathy, R.N. Mahapatra and R.B. Misra, "A method for evaluation of fuzzy reliability of multistage interconnection networks", *Comp. Sc. and Informatics Journal*, vol. 25(4), pp. 17-19, 1995.

[13] G.B. Adams, D.P. Agrawal and H.J. Siege, A Survey and comparison of fault-tolerant multistage interconnection networks, *IEEE computer*, 14-27, June,1987.